

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR
(AUTONOMOUS)

B.Tech. I Year I Semester Regular & Supplementary Examinations December/January-2024/2025

LINEAR ALGEBRA & CALCULUS

(Common to All)

Time: 3 Hours

Max. Marks: 70

PART-A

(Answer all the Questions 10 x 2 = 20 Marks)

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|---|--|-----|----|----|
| 1 | a Define rank of the matrix. | CO1 | L1 | 2M |
| | b What is the Consistency and Inconsistency of system of linear equations? | CO1 | L1 | 2M |
| | c Define Eigen values and Eigen vectors of a matrix. | CO2 | L1 | 2M |
| | d Find the symmetric matrix corresponding to the quadratic form $ax^2 + 2hxy + by^2$. | CO2 | L2 | 2M |
| | e Expand Taylor's series of the function $f(x)$ in powers of $(x - a)$. | CO3 | L2 | 2M |
| | f State Cauchy's mean value theorem. | CO3 | L1 | 2M |
| | g Evaluate $\lim_{x \rightarrow 1} \lim_{y \rightarrow 2} \frac{2x^2y}{x^2 + y^2 + 1}$ | CO5 | L5 | 2M |
| | h State Functional Dependence. | CO5 | L1 | 2M |
| | i Evaluate $\int_0^2 \int_0^x y dy dx$ | CO6 | L5 | 2M |
| | j Evaluate $\int_0^\pi \int_0^{\sin\theta} r dr d\theta$ | CO6 | L5 | 2M |

PART-B

(Answer all Five Units 5 x 10 = 50 Marks)

UNIT-I

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|---|--|-----|----|----|
| 2 | a Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ into Echelon form and find its rank. | CO1 | L3 | 5M |
| | b Find whether the following equations are consistent if so solve them $x + y + 2z = 4$; $2x - y + 3z = 9$; $3x - y - z = 2$. | CO1 | L3 | 5M |

OR

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|---|---|-----|----|-----|
| 3 | Solve the equations $3x + y + 2z = 3$; $2x - 3y - z = -3$; $x + 2y + z = 4$ Using Gauss elimination method. | CO1 | L3 | 10M |
|---|---|-----|----|-----|

UNIT-II

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|---|---|-----|----|-----|
| 4 | Determine the modal matrix P of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. Verify that $P^{-1}AP$ is a diagonal matrix. | CO2 | L2 | 10M |
|---|---|-----|----|-----|

OR

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|---|---|-----|----|----|
| 5 | a Show that the matrix $A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ satisfies its characteristic equation. | CO2 | L2 | 5M |
| | b State the nature of the Quadratic form $2x_1x_2 + 2x_1x_3 + 2x_2x_3$. | CO2 | L1 | 5M |

UNIT-III

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|---|--|-----|----|----|
| 6 | a Verify Lagrange's mean value theorem for $f(x) = \log_e x$ in $[1, e]$. | CO3 | L2 | 5M |
| | b Express the polynomial $2x^3 + 7x^2 + x - 6$ in power of $(x - 2)$ by Taylor's series. | CO4 | L3 | 5M |

OR

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|---|---|-----|----|----|
| 7 | a Show that $\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ by Maclaurin's theorem. | CO4 | L2 | 5M |
| | b Verify Rolle's Theorem for the function $f(x) = \frac{\sin x}{e^x}$ in $[0, \pi]$ | CO3 | L2 | 5M |

UNIT-IV

8 a If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, prove that $(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z})^2 U = \frac{-9}{(x+y+z)^2}$ CO5 L5 5M

b If $u = \tan^{-1}[\frac{2xy}{x^2-y^2}]$ then Prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. CO5 L5 5M

OR

9 a Find a point on the plane $3x + 2y + z - 12 = 0$, which is nearest to the origin. CO5 L1 5M

b Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest and farthest from the point $(3, 1, -1)$. CO5 L1 5M

UNIT-V

10 Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} (xy) dy dx$ and hence evaluate the same. CO6 L1 10M

OR

11 a Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ CO6 L1 5M

b Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z dz dx dy$ CO6 L5 5M

*** END ***

